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TIME-DEPENDENT RELIABILITY ANALYSIS FOR BIVARIATE RESPONSES

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ABSTRACT

Time-dependent system reliability is measured by the probability that the responses of a system do not exceed prescribed failure thresholds over a period of time. In this work, an efficient time-dependent reliability analysis method is developed for bivariate responses that are general functions of random variables and stochastic processes. The proposed method is based on single and joint upcrossing rates, which are calculated by the First Order Reliability Method (FORM). The method can efficiently produce accurate upcrossing rates for the systems with two responses. The upcrossing rates can then be used for system reliability predictions with two responses. As the general system reliability may be approximated with the results from reliability analyses for individual responses and bivariate responses, the proposed method can be extended to reliability analysis for general systems with more than two responses. Two examples, including a parallel system and a series system, are presented.

1. INTRODUCTION

Reliability is the ability that a component or system performs its intended function in routine circumstances for a given period of time. System reliability analysis is much more difficult than component reliability analysis. Many progresses have been made in system reliability analysis. For example, Ditlevsen [1] approximated the system reliability using a bounding formulas. Song and Kang [2] developed a matrix-based system reliability (MSR) method, which can calculate the system reliability and system parameter sensitivities by a convenient matrix-based framework. Nguyen [3] later developed a reliability-based system design optimization method by using the MSR method. Mahadevan [4] and Ambartzumian [5] proposed a system reliability method using a standard normal multivariate cumulative distribution function (CDF); by employing the Morgan's laws [6], the method expresses the system probability of failure as the intersection of a set of unions of subsystems. More system reliability analysis methods have been reported in [7].

Although reliability is defined for a period of time and is also a function of time, most of the aforementioned reliability methods are for time-invariant reliability which does not change over time. In many engineering applications, however, the limit-state function changes over time, because time appears explicitly in the function or stochastic processes are part of the input variables, or both. Examples include function generator mechanisms [8, 9], bridges under stochastic loading [10, 11], hydrokinetic turbine system subjected to wave or river flow loading [12, 13], and vehicles running on stochastic road surfaces [14].

Time-dependent reliability analysis is much more challenging than its time-independent counterpart. The most common time-dependent reliability method is the Rice formula [15, 16] developed in 1944 and is still widely used nowadays. There are many developments in time-dependent reliability in recent years. For instance, for component reliability problems, Mourelatos [17] employed the time-series modeling and importance sampling method to approximate the time-dependent reliability. Andrieu et al. [18] proposed a PHI2

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method for the time-dependent component reliability analysis for general problems with both random variables and stochastic processes. By using the Rice's formula [15, 16], Du and Hu [12, 19] developed a time-dependent reliability model for hydrokinetic turbine blades. Singh et al. [14] proposed the concept of composite limit-state function for time-dependent reliability analysis for a special group of problems. To improve the accuracy of time-dependent component reliability analysis, Du and Hu [20] proposed a joint upcrossing method based on the work of Madsen [21].

Studies on time-dependent system reliability have also been reported. For example, a method was developed to estimate the joint first-passage probability of failure for systems under stochastic excitation [22]. An approximation method was reported for estimating the conditional first passage probability of systems under modulated white noise excitation [23]. By combining Monte Carlo simulations with the asymptotic extreme value theory, Radhika [24] proposed a reliability analysis method for nonlinear vibrating systems. Some of the above methods have been verified to have good accuracy for systems subjected to multiple Gaussian stationary stochastic processes. These methods, however, cannot be directly applied to general problems where the input variables of a limit-state function contain time, random variables, and non-stationary stochastic processes.

In this work, a new time-dependent system reliability analysis method for bivariate responses is proposed for problems with nonlinear limit-state functions of time, random variables, and stochastic processes. The new method is an extension of the work in [22] and is based on the FORM and the Rice's formula. The major development is the derivations of bivariate joint upcrossing rates, which can be used for estimating time-dependent system reliability for bivariate responses. Since the bivariate joint probabilities are the basis for general system reliability analysis when the reliability bound method is used, the proposed method can also be applied to general time-dependent system reliability analysis for general systems with more than two components.

In Section 2, the background of time-dependent reliability is given. In Section 3, the upcrossing rate method is first introduced; equations are then derived for the bivariate joint upcrossing rates. The numerical procedure is summarized in Section 4, followed by two demonstration examples in Section 5. Conclusions are presented in Section 6.

2. TIME-DEPENDENT SYSTEM RELIABILITY

In this work a component corresponds to a failure mode. Suppose there are r failure modes or r components. For component i, where $i = 1, 2, \ldots, r$, let its limit-state function be $G_i = g_i(\mathbf{X}, \mathbf{Y}(t), t)$, where $\mathbf{X} = [X_1, X_2, \cdots, X_n]$ is a vector of random variables, $\mathbf{Y}(t) = [Y_1(t), Y_2(t), \cdots, Y_m(t)]$ is a vector of stochastic processes, and G_i is the response variable. t stands for time.

The time-dependent probability of failure $p_{f,i}(t_0, t_s)$ of component i over the time interval $[t_0, t_s]$ is defined by

$$p_{f,i}(t_0, t_s) = \Pr\{g_i(\mathbf{X}, \mathbf{Y}(t), t) > e_i, \exists t \in [t_0, t_s]\} \tag{1}$$

in which e_i is the failure threshold, and $\Pr\{\cdot\}$ stands for a probability.

Let Ω_s be the safe region for a system. For a series system,

$$\Omega_s = \left\{ [\mathbf{X}, \mathbf{Y}(t)] \Big| \bigcap_{i=1}^{r} g_i(\mathbf{X}, \mathbf{Y}(t), t) < e_i, \forall t \in [t_0, t_s] \right\} \tag{2}$$

in which "$\cap$" stands for an intersection.

For a parallel system,

$$\Omega_s = \left\{ [(\mathbf{X}, \mathbf{Y}(t)] \Big| \bigcup_{i=1}^{r} g_i(\mathbf{X}, \mathbf{Y}(t), t) < e_i, \forall t \in [t_0, t_s] \right\} \tag{3}$$

in which "$\cup$" stands for a union.

With above definitions, the time-dependent system reliability $R_s(t_0, t_s)$ is given by

$$R_s(t_0, t_s) = \Pr\{[\mathbf{X}, \mathbf{Y}(t)] \in \Omega_s, \forall t \in [t_0, t_s]\} \tag{4}$$

The system reliability requires not only the component reliability but also joint probabilities up to an order of r. Evaluating a joint probability with a high order is extremely difficult. To make the system reliability easier, Ditlevsen [1] proposed a bound formula for a series system. In the bound formula, the system probability of failure is bounded by functions of component probability of failure $p_{f,i}(t_0, t_s)$ and bivariate probability of failure $p_{f,ij}(t_0, t_s)$.

As reviewed previously, many time-dependent reliability methods are available for $p_{f,i}(t_0, t_s)$. In this work, we develop a new method for the bivariate probability of failure $p_{f,ij}(t_0, t_s)$.

3. TIME-DEPENDENT RELIABILITY FOR BIVARIATE RESPONSES

For limit-state functions $G_i = g_i(\mathbf{X}, \mathbf{Y}(t), t)$ and $G_j = g_j(\mathbf{X}, \mathbf{Y}(t), t)$, the joint time-dependent probability of failure is given by

$$\begin{aligned} p_{f,ij}(t_0, t_s) = \Pr\{ & g_i(\mathbf{X}, \mathbf{Y}(\chi), \chi) > e_i \cap \\ & g_j(\mathbf{X}, \mathbf{Y}(\tau), \tau) > e_j, \exists \chi \text{ and } \exists \tau \in [t_0, t_s]\} \end{aligned} \tag{5}$$

Eq. (5) is further transformed into [22]

$$\begin{aligned} p_{f,ij}(t_0, t_s) = & \Pr\{g_i(\mathbf{X}, \mathbf{Y}(\chi), \chi) > e_i, \exists \chi \in [t_0, t_s]\} \\ & + \Pr\{g_j(\mathbf{X}, \mathbf{Y}(\tau), \tau) > e_j, \exists \tau \in [t_0, t_s]\} \\ & - \Pr\{g_i(\mathbf{X}, \mathbf{Y}(\chi), \chi) > e_i \cup \\ & g_j(\mathbf{X}, \mathbf{Y}(\tau), \tau) > e_j, \exists \chi \text{ and } \tau \in [t_0, t_s]\} \end{aligned} \tag{6}$$

The first two terms on the right-hand side of Eq. (6) are component probabilities of failure, and Eq. (6) can be rewritten as

$$p_{f,ij}(t_0, t_s) = p_{f,i}(t_0, t_s) + p_{f,j}(t_0, t_s) - p_{f,i\cup j}(t_0, t_s) \tag{7}$$

where

$$p_{f,i\cup j}(t_0,t_s)=\Pr\{g_i(\mathbf{X},\mathbf{Y}(\chi),\chi)>e_i \cup g_j(\mathbf{X},\mathbf{Y}(\tau),\tau)>e_j, \exists\chi \text{ and } \tau\in[t_0,t_s]\} \quad (8)$$

which can be considered as the time-dependent probability of failure for a series system with components *i* and *j*. In the following sections, we first discuss the time-dependent component reliability. We then derive equations for time-dependent joint probability, $p_{f,i\cup j}(t_0,t_s)$.

3.1 Time-dependent component reliability analysis

In this work, we employ the upcrossing rate method [18] to evaluate the time-dependent component probability of failure.

3.1.1 Upcrossing rate method for time-dependent component reliability analysis

For a general limit-state function $G_k=g_k(\mathbf{X},\mathbf{Y}(t),t), k=i \text{ or } j$ with threshold e_k, the time-dependent probability of failure $p_{f,k}(t_0,t_s)$ is given by the upcrossing rate method as follows:

$$p_{f,k}(t_0,t_s)=1-[1-p_{f,k}(t_0)]\exp\left\{-\int_{t_0}^{t_s}v_k^+(t)dt\right\} \quad (9)$$

in which $v_k^+(t)$ is the upcrossing rate of component *k* at time instant *t*, and $p_{f,k}(t_0)$ is the instantaneous probability of failure at t_0, given by

$$p_{f,k}(t_0)=\Pr\{g_k(\mathbf{X},\mathbf{Y}(t_0),t_0)>e_k\} \quad (10)$$

An upcrossing event happens when the response variable G_k passes the threshold e_k at time instant *t* from the safe region $G_k(t)<e_k$ to the failure region $G_k(t+\Delta t)>e_k$, where Δt is an infinitesimally small time interval. $v_k^+(t)$ is defined by

$$v_k^+(t)=\lim_{\Delta t\to 0}\frac{\Pr\{[g_k(\mathbf{X},\mathbf{Y}(t),t)<e_k]\cap[g_k(\mathbf{X},\mathbf{Y}(t+\Delta t),t+\Delta t)>e_k]\}}{\Delta t} \quad (11)$$

Eq. (9) is derived based on the assumption that all the upcrossings over $[t_0,t_s]$ are independent. Knowing $v_k^+(t)$, the component probability of failure, one can easily obtain $p_{f,k}(t_0,t_s)$ using Eq. (9).

The other commonly used method is FORM. Next, we will discuss the linearization in FORM and its use in estimating $v_k^+(t)$. We will also discuss how to derive equations for $p_{f,i\cup j}(t_0,t_s)$ by using the linearization.

3.1.2 Transformation of limit-state functions

FORM transforms random variables $\mathbf{X}$ and stochastic processes $\mathbf{Y}(t)$ into standard normal random variables $\mathbf{U}(t)=(\mathbf{U}_\mathbf{X},\mathbf{U}_\mathbf{Y}(t))$. Then the limit-state function becomes

$$\begin{aligned}G_k&=g_k(\mathbf{X},\mathbf{Y}(t),t)=g_k(T(\mathbf{U}_\mathbf{X}),T(\mathbf{U}_\mathbf{Y}(t)),t)\\&=g_k(\mathbf{U}(t),t), k=i \text{ or } j\end{aligned} \quad (12)$$

where $T(\cdot)$ stands for the transforming operator.

Then the MPP $\mathbf{u}_k^*(t)=(\mathbf{u}_\mathbf{X}^*,\mathbf{u}_\mathbf{Y}^*(t)), k=i \text{ or } j$ is found with the following optimization model

$$\begin{cases}\min_{\mathbf{u}}\|\mathbf{u}(t)\|\\ \text{subject to } g_k(\mathbf{u}(t),t)=e_k, k=i \text{ or } j\end{cases} \quad (13)$$

in which $\|\cdot\|$ stands for the determinant of a vector.

After the limit-state function is linearized at the MPP, the failure event $G_k=g_k(\mathbf{X},\mathbf{Y}(t),t)>e_k, k=i \text{ or } j$ becomes equivalent to the following event

$$L_k(t)=\boldsymbol{\alpha}_k(t)\mathbf{U}(t)^T>\beta_k(t), k=i \text{ or } j \quad (14)$$

in which

$$\beta_k(t)=\|\mathbf{u}_k^*(t)\|, k=i \text{ or } j \quad (15)$$

$$\boldsymbol{\alpha}_k(t)=-\mathbf{u}_k^*(t)\big/\|\mathbf{u}_k^*(t)\|, k=i \text{ or } j \quad (16)$$

$\beta_k(t), k=i \text{ or } j$ is called the Hasofer-Lind reliability index.

Therefore, failure events given in Eqs. (1) and (8) become

$$p_{f,i}(t_0,t_s)=\Pr\{L_i(\chi)=\boldsymbol{\alpha}_i(\chi)\mathbf{U}^T(\chi)>\beta_i(\chi), \exists\chi\in[t_0,t_s]\} \quad (17)$$

and

$$p_{f,i\cup j}(t_0,t_s)=\Pr\begin{Bmatrix}\boldsymbol{\alpha}_i(\chi)\mathbf{U}^T(\chi)>\beta_i(\chi), \exists\,\chi\in[t_0,t_s]\\ \cup\boldsymbol{\alpha}_j(\tau)\mathbf{U}^T(\tau)>\beta_j(\tau), \exists\,\tau\in[t_0,t_s]\end{Bmatrix} \quad (18)$$

In the next section, we will discuss the method for the approximation of the bivariate probability $p_{f,i\cup j}(t_0,t_s)$.

3.2 Time-dependent joint probability $p_{f,i\cup j}(t_0,t_s)$

3.2.1 Outcrossing rate method for time-dependent joint probability analysis

We now derive equations for the bivariate joint probability $p_{f,i\cup j}(t_0,t_s)$. With the same strategy of upcrossing rate in Eq. (9), $p_{f,i\cup j}(t_0,t_s)$ is given by

$$p_{f,i\cup j}(t_0,t_s)=1-R_{ij}(t_0)\exp\left\{-\int_{t_0}^{t_s}v_{i\cup j}^+(t)dt\right\} \quad (19)$$

in which $R_{ij}(t_0)$ is the probability that both components are safe at the initial time and is given by

$$R_{ij}(t_0)=\Pr\{g_i(\mathbf{X},\mathbf{Y}(t_0),t_0)\le e_i\cap g_j(\mathbf{X},\mathbf{Y}(t_0),t_0)\le e_j\} \quad (20)$$

$v_{i\cup j}^+(t)$ is the outcrossing rate of a series system with components *i* and *j* at time instant *t*. An outcrossing event occurs when the system outcrosses its bounds at time instant *t* from the safe region to the failure region. Fig. 1 shows three representative outcrossing events of the series system. For the outcrossing events, both components *i* and *j* are in the safe region at time instants t_m, *m=1, 2,* and *3*. The system then outcrosses into the failure region as a result of the upcrossing of G_i, or upcorring of G_j, or both the upcrossings of G_i and G_j at the following time instants, $t_m+\Delta t$, *m=1, 2,* and *3*. Given in mathematical form, the outcrossing rate $v_{i\cup j}^+(t)$ is given by the following limit:

$$v_{i\cup j}^{+}(t)=\lim_{\Delta t\to 0}\frac{\Pr\left\{\begin{matrix}\left[G_i(t)<e_i\cap G_j(t)<e_j\right]\cap\\ \left[G_i(t+\Delta t)>e_i\cup G_j(t+\Delta t)>e_j\right]\end{matrix}\right\}}{\Delta t}\qquad(21)$$

where Δt is an infinitesimally small time interval.

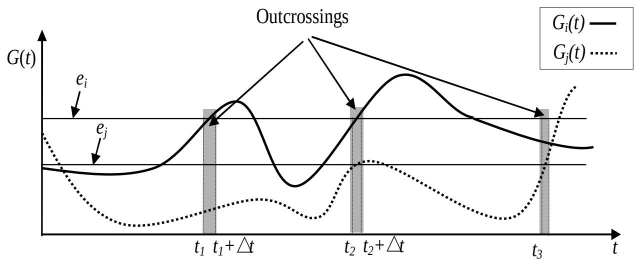

Fig. 1 Outcrossing events of a system with bivariate responses

The probability in Eq. (21) can be decomposed into three components.

$$\begin{aligned}&\Pr\left\{\begin{matrix}\left[G_i(t)<e_i\cap G_j(t)<e_j\right]\\ \cap\left[G_i(t+\Delta t)>e_i\cup G_j(t+\Delta t)>e_j\right]\end{matrix}\right\}\\ &=p_{ij}^{+-}(t)+p_{ij}^{-+}(t)+p_{ij}^{++}(t)\end{aligned}\qquad(22)$$

where

$$p_{ij}^{+-}(t)=\Pr\left\{\begin{matrix}\left[G_i(t)<e_i\cap G_j(t)<e_j\right]\\ \cap\left[G_i(t+\Delta t)>e_i\cap G_j(t+\Delta t)<e_j\right]\end{matrix}\right\}\qquad(23)$$

$$p_{ij}^{-+}(t)=\Pr\left\{\begin{matrix}\left[G_i(t)<e_i\cap G_j(t)<e_j\right]\\ \cap\left[G_i(t+\Delta t)<e_i\cap G_j(t+\Delta t)>e_j\right]\end{matrix}\right\}\qquad(24)$$

$$p_{ij}^{++}(t)=\Pr\left\{\begin{matrix}\left[G_i(t)<e_i\cap G_j(t)<e_j\right]\\ \cap\left[G_i(t+\Delta t)>e_i\cap G_j(t+\Delta t)>e_j\right]\end{matrix}\right\}\qquad(25)$$

$p_{ij}^{+-}(t)$ is the probability that $G_i(t)$ upcrosses its barrier e_i while $G_j(t)$ remains below its barrier e_j at t, $p_{ij}^{-+}(t)$ is the probability that $G_j(t)$ upcrosses its barrier e_j while $G_i(t)$ remains below its barrier e_i at t, and $p_{ij}^{++}(t)$ is the probability that both $G_i(t)$ and $G_j(t)$ upcross their barriers at t.

Three corresponding joint upcrossing rates are then defined by

$$v_{ij}^{+-}(t)=\lim_{\Delta t\to 0}\left(p_{ij}^{+-}(t)\big/\Delta t\right)\qquad(26)$$

$$v_{ij}^{-+}(t)=\lim_{\Delta t\to 0}\left(p_{ij}^{-+}(t)/\Delta t\right)\qquad(27)$$

$$v_{ij}^{++}(t)=\lim_{\Delta t\to 0}\left(p_{ij}^{++}(t)\big/\Delta t\right)\qquad(28)$$

Then

$$v_{i\cup j}^{+}(t)=v_{ij}^{+-}(t)+v_{ij}^{-+}(t)+v_{ij}^{++}(t)\qquad(29)$$

Equations for $v_{ij}^{+-}(t)$ are available for special limit-state functions with stationary Gaussian vector processes [22]. In the subsequent subsections, we will derive equations for $v_{ij}^{+-}(t)$ and other two joint upcrossing rates for general limit-state functions. The derivations are based on the approximation discussed in Sec. 3.1.2.

3.2.2 $v_{ij}^{+-}(t)$

Substituting Eqs. (17) into Eq. (23) yields

$$p_{ij}^{+-}(t)=\Pr\left\{\begin{matrix}\left[L_i(t)<\beta_i(t)\cap L_j(t)<\beta_j(t)\right]\\ \cap\left[L_i(t+\Delta t)>\beta_i(t+\Delta t)\cap L_j(t+\Delta t)<\beta_j(t+\Delta t)\right]\end{matrix}\right\}\qquad(30)$$

It is the probability that $L_i(t)$ upcrosses its barrier $\beta_i(t)$ while $L_j(t)$ remains below its barrier $\beta_j(t)$ at t. With the Rice's formula [15, 16], $v_{ij}^{+-}(t)=\lim_{\Delta t\to 0}p_{ij}^{+-}(t)\big/\Delta t$ can be calculated by the following integral:

$$v_{ij}^{+-}(t)=\int_{-\infty}^{\beta_j(t)}\int_{\dot\beta_i(t)}^{\infty}[\dot l_i-\dot\beta_i(t)]f_{L_iL_j\dot L_i}(\beta_i(t),l_j,\dot l_i)d\dot l_i dl_j\qquad(31)$$

where $f_{L_iL_j\dot L_i}(\cdot,\cdot,\cdot)$ is the joint PDF of $L_i(t)$, $L_j(t)$, and $\dot L_i(t)$.

As no close form expression of $v_{ij}^{+-}(t)$ is available, transformations for Eq. (31) are required. Based on the transformation, Eq. (31) is rewritten as below [22]:

$$v_{ij}^{+-}(t)=\phi(\beta_i(t))\int_{-\infty}^{\beta_j(t)}\frac{\sigma_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}{\sigma_{L_j|L_i=\beta_i(t)}}\phi\left(\frac{l_j-\mu_{L_j|L_i=\beta_i(t)}}{\sigma_{L_j|L_i=\beta_i(t)}}\right)Hdl_j\qquad(32)$$

where

$$\begin{aligned}H&=\phi\left(\frac{\dot\beta_i(t)-\mu_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}{\sigma_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}\right)\\ &-\frac{\dot\beta_i(t)-\mu_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}{\sigma_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}\Phi\left(\frac{\dot\beta_i(t)-\mu_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}{\sigma_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}}\right)\end{aligned}\qquad(33)$$

The above equations indicate that $\mu_{Lj|Li=\beta_i}$, $\sigma_{Lj|Li=\beta_i}$, $\mu_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}$, and $\sigma_{\dot L_i|L_i=\beta_i(t),\,L_j=l_j}$ are required to solve for $v_{ij}^{+-}(t)$, for which the mean and covariance of $\dot L_i(t)$ and $\mathbf{L}=[L_i(t),L_j(t)]$ must be obtained.

Since $L_i(t)=\boldsymbol{\alpha}_i(t)\mathbf{U}^T(t)=\boldsymbol{\alpha}_{\mathbf{X}i}(t)\mathbf{U}_{\mathbf{X}}^T+\boldsymbol{\alpha}_{\mathbf{Y}i}(t)\mathbf{U}_{\mathbf{Y}}^T(t)$, $\dot L_i(t)$ is given by

$$\dot L_i(t)=\dot{\boldsymbol{\alpha}}_i(t)\mathbf{U}^T(t)+\boldsymbol{\alpha}_{\mathbf{Y}i}(t)\dot{\mathbf{U}}_{\mathbf{Y}}^T(t)\qquad(34)$$

With Eqs. (17) and (34), the covariance matrix of $\dot L$ and $\mathbf{L}$ are given as below.

$$\mathbf{c}_{\dot{\mathbf{L}}}=\begin{bmatrix}\mathbf{c}_{\dot L_i\dot L_i} & \mathbf{c}_{\dot L_i\mathbf{L}}\\ \mathbf{c}_{\mathbf{L}\dot L_i} & \mathbf{c}_{\mathbf{LL}}\end{bmatrix}=\begin{bmatrix}\omega_i^2(t) & c_{L_i\dot L_i} & c_{L_j\dot L_i}\\ c_{L_i\dot L_i} & 1 & c_{L_iL_j}\\ c_{L_j\dot L_i} & c_{L_iL_j} & 1\end{bmatrix}\qquad(35)$$

in which $\omega_i^2(t) = \dot{\boldsymbol{\alpha}}_i(t)\dot{\boldsymbol{\alpha}}_i^T(t) + \boldsymbol{\alpha}_i(t)\ddot{\mathbf{C}}_{12}(t,t)\boldsymbol{\alpha}_i^T(t)$ is provided in Ref. [20], $\ddot{\mathbf{C}}_{12}$ is the second order partial derivative of covariance matrix of $\mathbf{U}(t)$, and the other components of the matrix are given below.

$$c_{L_i\dot{L}_i} = \boldsymbol{\alpha}_i(t)\dot{\boldsymbol{\alpha}}_i^T(t) \tag{36}$$

$$c_{L_iL_j} = \boldsymbol{\alpha}_i(t)\boldsymbol{\alpha}_j^T(t) \tag{37}$$

$$c_{L_j\dot{L}_i} = \dot{\boldsymbol{\alpha}}_i(t)\boldsymbol{\alpha}_j^T(t) \tag{38}$$

With the covariance matrix $\mathbf{c}_{\mathbf{L}}$, the conditional means and standard deviations are now available. They are given by

$$\mu_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j} = \mathbf{c}_{\dot{L}_i\mathbf{L}}\mathbf{c}_{\mathbf{LL}}{}^{-1}\mathbf{l} \tag{39}$$

$$\sigma^2_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j} = \mathbf{c}_{\dot{L}_i\dot{L}_i} - \mathbf{c}_{\dot{L}_i\mathbf{L}}\mathbf{c}_{\mathbf{LL}}{}^{-1}\mathbf{c}_{\mathbf{L}\dot{L}_i} \tag{40}$$

in which $\mathbf{l} = [\beta_i(t); l_j]$.

Substituting Eq. (35) into Eqs. (39) and (40) yields

$$\mu_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j} = \frac{\beta_i(t)(c_{L_j\dot{L}_i}c_{L_iL_j} - c_{L_i\dot{L}_i}) + l_j(c_{L_i\dot{L}_i}c_{L_iL_j} - c_{L_j\dot{L}_i})}{c^2{}_{L_iL_j} - 1} \tag{41}$$

$$\begin{aligned}&\sigma^2{}_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j}\\ &= \omega_i^2(t) - \frac{c_{L_i\dot{L}_i}(c_{L_j\dot{L}_i}c_{L_iL_j} - c_{L_i\dot{L}_i}) + c_{L_j\dot{L}_i}(c_{L_i\dot{L}_i}c_{L_iL_j} - c_{L_j\dot{L}_i})}{c^2{}_{L_iL_j} - 1}\end{aligned} \tag{42}$$

Since $\boldsymbol{\alpha}_i(t)\boldsymbol{\alpha}_i^T(t) = 1$, we obtain that $c_{L_i\dot{L}_i} = \boldsymbol{\alpha}_i(t)\dot{\boldsymbol{\alpha}}_i^T(t) = 0$. Eqs. (41) and (42) are then simplified as

$$\mu_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j} = (\beta_i(t)c_{L_j\dot{L}_i}c_{L_iL_j} - l_jc_{L_j\dot{L}_i}) \big/ (c^2_{L_iL_j} - 1) \tag{43}$$

and

$$\sigma^2_{\dot{L}_i|L_i=\beta_i(t),\,L_j=l_j} = \omega_i^2(t) + c^2_{L_j\dot{L}_i} \big/ (c^2_{L_iL_j} - 1) \tag{44}$$

Similarly,

$$\mu_{Lj|Li=\beta_i(t)} = c_{L_iL_j}\beta_i(t) \tag{45}$$

$$\sigma^2_{L_j|L_i=\beta_i(t)} = 1 - c^2_{L_iL_j} \tag{46}$$

So far all the equations needed to calculate $v_{ij}^{+-}(t)$ are derived. They can be solved by substituting Eqs. (41) through (46) into Eq. (32).

3.2.3 $v_{ij}^{-+}(t)$

After obtaining the first joint upcrossing rate $v_{ij}^{+-}(t)$, we can easily obtain the second upcrossing rate $v_{ij}^{-+}(t)$. We just simply switch the subscripts *i* and *j* in Eqs. (30) through (46).

3.2.4 $v_{ij}^{++}(t)$

Substituting Eqs. (17) into Eq. (25) yields

$$v_{ij}^{++}(t) = \lim_{\Delta t \to 0}\left(M_{ij}^{++} \big/ \Delta t\right) \tag{47}$$

where

$$\begin{aligned}M_{ij}^{++} = \Pr[&L_i(t) < \beta_i(t) \cap L_j(t) < \beta_j(t)]\\ &\cap [L_i(t+\Delta t) > \beta_i(t+\Delta t) \cap L_j(t+\Delta t) > \beta_j(t+\Delta t)]\end{aligned} \tag{48}$$

Defining $Z_i(t) = L_i(t) - \beta_i(t)$, we have

$$M_{ij}^{++} = \int_0^{+\infty}\int_0^{+\infty}\int_{-\infty}^0\int_{-\infty}^0 f_{Z_iZ_j\dot{Z}_i\dot{Z}_j}(z_i, z_j, \dot{z}_i, \dot{z}_j)dz_idz_jd\dot{z}_id\dot{z}_j \tag{49}$$

t is omitted in Eq. (49) for brevity; for example z_i stands for $z_i(t)$ now. Appendix A shows that $v_{ij}^{++}(t)$ is zero when Δt becomes infinitely small. Therefore, $v_{ij}^{++}(t) = 0$.

Having obtained all the three joint upcrossing rates, we obtain the outcrossing rate in Eq. (29) as follows:

$$v_{i\cup j}^{+}(t) = v_{ij}^{+-}(t) + v_{ij}^{-+}(t) \tag{50}$$

3.2.5 $R_{ij}(t_0)$

$R_{ij}(t_0)$ is another component needed for the system reliability analysis. After the MPPs of components *i* and *j* are found at t_0, $R_{ij}(t_0)$ is calculated by [4]

$$R_{ij}(t_0) = \Phi(\beta_i(t_0), \beta_j(t_0), \rho_0) \tag{51}$$

in which $\Phi(\cdot,\cdot,\cdot)$ is the CDF of a bivariate normal random variable, and ρ_0 is the coefficient of correlation between the two components. ρ_0 is given by

$$\rho_0 = \boldsymbol{\alpha}_i(t_0)\boldsymbol{\alpha}_j^T(t_0) \tag{52}$$

With Eqs. (19) through (52), $p_{f,i\cup j}(t_0, t_s)$ can be estimated. Then, the time-dependent probability of failure for bivariate responses, $p_{f,ij}(t_0, t_s)$, can be computed.

3.3 System reliability analysis

With the availability of the outcrossing rate $v_{i\cup j}^{+}(t)$, we now summarize the proposed system reliability analysis method for bivariate responses.

For a series system with component *i* and *j*, the system probability of failure is given by

$$p_{f,s}(t_0, t_s) = p_{f,i\cup j}(t_0, t_s) = 1 - R_{ij}(t_0)\exp\left\{-\int_{t_0}^{t_s} v_{i\cup j}^{+}(t)dt\right\} \tag{53}$$

For a parallel system,

$$\begin{aligned}&p_{f,s}(t_0, t_s) = 1 - R_i(t_0)\exp\left\{-\int_{t_0}^{t_s} v_i^{+}(t)dt\right\}\\ &-R_j(t_0)\exp\left\{-\int_{t_0}^{t_s} v_j^{+}(t)dt\right\} + R_{ij}(t_0)\exp\left\{-\int_{t_0}^{t_s} v_{i\cup j}^{+}(t)dt\right\}\end{aligned} \tag{54}$$

Until now all the equations needed for the time-dependent system reliability analysis for bivariate responses are available.

4. NUMERICAL PROCEDURE

A flowchart for the proposed method is provided in Fig. 2; The main steps are summarized as below.

- Step 1: Initialization of parameters - Transform uncertain variables into standard Gaussian random variables and stochastic processes.
- Step 2: FORM - Perform the MPP search at time instant *t* using Eq. (13); obtain the associated reliability indexes, and the derivative of reliability indexes.

- Step 3: Initial reliability - Calculate the initial component reliability using Eq. (9) and initial system reliability using Eqs. (51) and (52).
- Step 4: Upcrossing rates and outcrossing rate - Compute $v_{ij}^{+-}(t)$ and $v_{ij}^{-+}(t)$ using Eq.(32); then obtain the joint upcrossing rate $v_{i\cup j}^{+}(t)$.
- Step 5: Integration - Integrate the upcrossing rates over $[t_0, t_s]$.
- Step 6: System reliability - Obtain the system probability of failure $p_{f,s}(t_0, t_s)$ using Eq. (53) or (54).

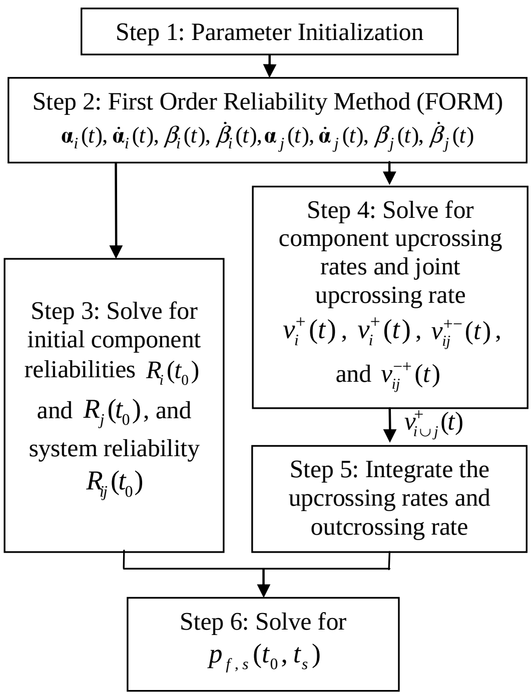

Fig. 2 Flowchart of the proposed method

5. NUMERICAL EXAMPLES

In this section, we use two examples to demonstrate the proposed method. They are a Daniels system [25] and a function generator mechanism system.

5.1 Example 1 – A Daniels System

Fig. 3 shows a structural system under stochastic loading. The system consists of two bars. Due to different manufacturing precisions, the two bars have different standard deviations in their dimensions. As the two bars are exposed to corrosions, their widths and heights decrease at the rates of k_1 and k_2, respectively. Each of the two bars resists a load of $P(t)/2$ until both of the two bars yield. The task is to determine the time-dependent system probabilities of failure over different time intervals up to [0, 20] years.

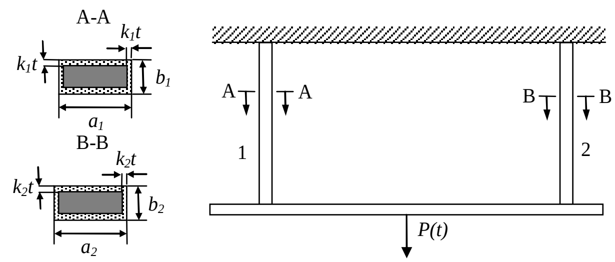

Fig. 3 A two-bar system

Since the system is parallel, the time-dependent system probability of failure is given by

$$p_{f,s}(t_0, t_s) = \Pr\{g_1(\mathbf{X}, \mathbf{Y}(\chi), \chi) > e_1 \cap g_2(\mathbf{X}, \mathbf{Y}(\tau), \tau) > e_2, \exists \chi \text{ and } \tau \in [t_0, t_s]\} \tag{55}$$

where

$$g_i(\mathbf{X}, \mathbf{Y}(t), t) = P(t)/2 - (a_i - 2k_i t)(b_i - 2k_i t)\sigma_{bi} \tag{56}$$

and $i = 1, 2$, k_1 =5×10^{-4} in/year, k_2 =3×10^{-4} in/year, e_1 =0, e_2 =0, $\mathbf{X} = [a_1, b_1, a_2, b_2, \sigma_{b1}, \sigma_{b2}]$, and $\mathbf{Y}(t) = [P(t)]$; σ_{b1} and σ_{b2} are the yield strengths of bars 1 and 2, respectively. The parameters in Eqs. (55) and (56) are presented in Table 1.

Table 1 Variables in Example 1

Variable	Mean	Standard deviation	Distribution	Autocorrelation
a_1	1.3 in	0.01 in	Gaussian	N/A
b_1	1.2 in	0.01 in	Gaussian	N/A
a_2	1.3 in	0.05 in	Gaussian	N/A
b_2	1.2 in	0.05 in	Gaussian	N/A
σ_{b1}	36 kpsi	0.36 kpsi	Gaussian	N/A
σ_{b2}	36 kpsi	0.36 kpsi	Gaussian	N/A
$P(t)$	90 kpsi	9 kpsi	Gaussian Process	Eq. (57)

The auto-correlation function of the stochastic process $P(t)$ is given by

$$\rho^P(t_1, t_2) = \exp\left[-(t_2 - t_1)^2 / \zeta^2\right] \tag{57}$$

where $\zeta = 2$ years is the correlation length. The longer is the time interval $t_2 - t_1$, the weaker is the auto-correlation.

To evaluate the accuracy of the new method, Monte Carlo simulation (MCS) is performed using a large sample size of 10^7. The upcrossing rates $v_{12}^{+-}(t)$, $v_{12}^{-+}(t)$, and $v_{1\cup 2}^{+}(t)$ obtained from the proposed method are also compared with MCS.

Figs. 4 through 6 depict the upcrossing rates $v_{12}^{+-}(t)$, $v_{12}^{-+}(t)$, and outcrossing rate $v_{1\cup 2}^{+}(t)$ from both the new method and MCS.

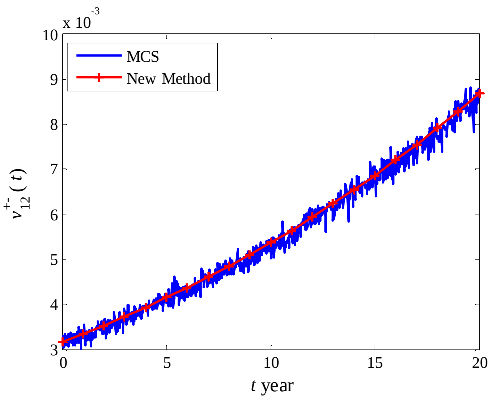

Fig. 4 $v_{12}^{+-}(t)$ over time interval [0, 20] years

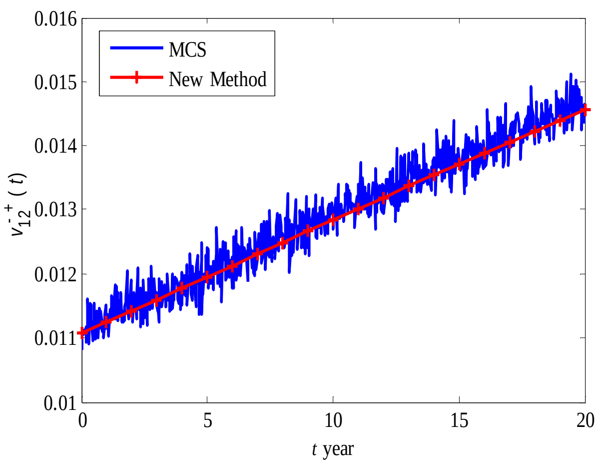

Fig. 5 $v_{12}^{-+}(t)$ over time interval [0, 20] years

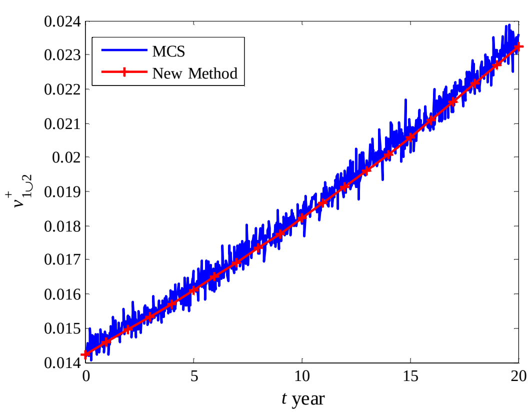

Fig. 6 $v_{1\cup 2}^{+}(t)$ over time interval [0, 20] years

Note that the curves of upcrossing rates and outcrossing rate from MCS are not smooth. The noise comes from the numerical discretization of stochastic process. Nevertheless, the results show the good consistency between the MCS results and those from the proposed method. This example indicates that the proposed method can produce accurate joint upcrossing rates and outcrossing rate that are needed for time-dependent system reliability analysis.

Using the outcrossing rate $v_{1\cup 2}^{+}(t)$, the system reliability analysis result is obtained. The time-dependent system probability of failure $p_{f,s}(t_0, t_s)$ is depicted in Figs. 7 and given in Table 2.

As shown in Fig. 7, the error of the new method becomes larger with a longer period of time or with a larger probability of failure. The error resource is mainly the assumption of independent crossings. It is the intrinsic drawback of the upcrossing and outcrossing rate method [21].

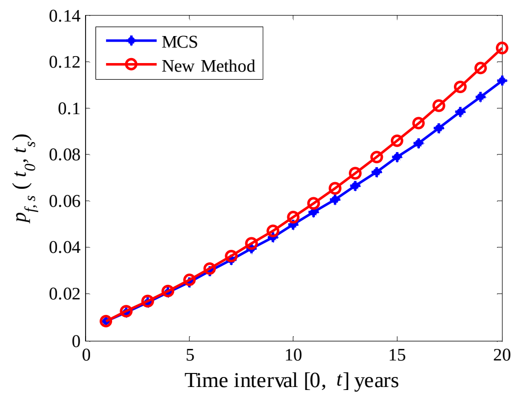

Fig. 7 $p_{f,s}(t_0, t_s)$ time interval [0, 20] years

Table 2 Time-dependent system probability of failure

Time interval	New Method		MCS	
	$p_{f,s}(t_0, t_s)$	ε (%)	$p_{f,s}(t_0, t_s)$	95% confidence interval
[0, 2]	0.0126	2.64	0.0123	[0.0121, 0.0125]
[0, 5]	0.0261	4.46	0.0250	[0.0247, 0.0253]
[0, 8]	0.0416	5.31	0.0395	[0.0391, 0.0398]
[0, 11]	0.0591	7.08	0.0552	[0.0547, 0.0556]
[0, 14]	0.0789	8.21	0.0729	[0.0724, 0.734]
[0, 17]	0.1010	10.03	0.0918	[0.0912, 0.0924]
[0, 20]	0.1256	11.98	0.1122	[0.1116, 0.1128]

5.2 Example 2 – A function generator mechanism system

A function generator mechanism is a mechanism used to realize a desired motion [8, 9]. Such a system is shown in Fig. 8. This system consists of two function generator mechanisms. Mechanism 1, a four-bar linkage mechanism with links B_1, B_2, B_3 and B_4, generates a sine function while mechanism 2, the other four-bar linkage mechanism with links B_1, B_5, B_6 and B_7, generates a logarithm function.

For the sine function generator (Mechanism 1), the motion input and motion output are γ and $\kappa = \kappa_a(\mathbf{B}, \gamma)$, respectively. The required motion output is given by

$$\kappa_d(\gamma) = 60^\circ + 60^\circ \sin\left[\frac{3}{4}(\gamma - 97^\circ)\right] \tag{58}$$

For the logarithm function generator (Mechanism 2), the motion input and motion output are θ and $\eta = \eta_a(\mathbf{B}, \theta)$, respectively. The required motion output is given by

$$\eta_d(\theta) = 60^\circ \left(\log_{10}[(\theta + 15^\circ)/60^\circ]/\log_{10}(2)\right) \tag{59}$$

A motion error is the difference between the actual motion output and the required motion output. For the two mechanisms, their motion errors are

$$\varepsilon_\kappa(\mathbf{B}, \gamma) = \kappa_a(\mathbf{B}, \gamma) - \kappa_d(\gamma) \tag{60}$$

and

$$\varepsilon_\eta(\mathbf{B}, \theta) = \eta_a(\mathbf{B}, \theta) - \eta_d(\theta) \tag{61}$$

where $\mathbf{B} = [B_1, B_2, \cdots, B_7]$.

Links B_2 and B_5 are welded together, the two input angles satisfy

$$\gamma = 62^\circ + \theta \tag{62}$$

From the mechanism analysis, the following equations can be obtained:

$$\kappa_a(\mathbf{B}, \gamma) = 2\arctan\left(\frac{-E_\kappa \pm \sqrt{E_\kappa^2 + D_\kappa^2 - F_\kappa^2}}{F_\kappa - D_\kappa}\right) \tag{63}$$

where $D_\kappa = 2B_4(B_1 - B_2\cos\gamma)$, $E_\kappa = -2B_2B_4\sin\gamma$, and $F_\kappa = B_1^2 + B_2^2 + B_4^2 - B_3^2 - 2B_1B_2\cos\gamma$.

$$\eta_a(\mathbf{B}, \theta) = 2\arctan\left(\frac{-E_\eta \pm \sqrt{E_\eta^2 + D_\eta^2 - F_\eta^2}}{F_\eta - D_\eta}\right) \tag{64}$$

$$\eta_d(\theta) = 60^\circ \log_{10}[(\theta + 15^\circ)/60^\circ]/\log_{10}(2) \tag{65}$$

where $D_\eta = 2B_7(B_1 - B_5\cos\theta)$, $E_\eta = -2B_5B_7\sin\theta$, and $F_\eta = B_1^2 + B_5^2 + B_7^2 - B_6^2 - 2B_1B_5\cos\theta$.

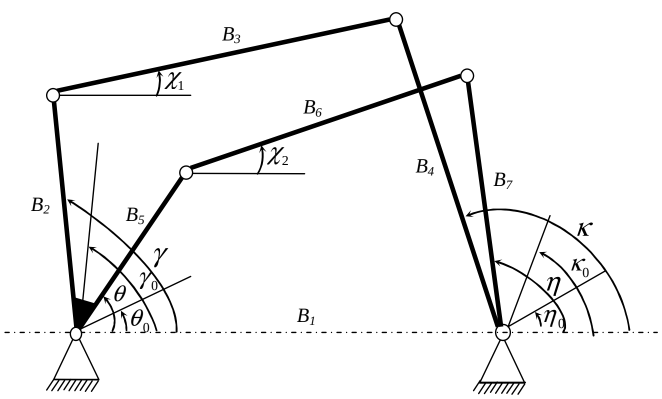

Fig. 8 A function generator mechanism system

In this problem, the time factor is the input angle θ. There are no stochastic processes in the input variables. The vector of random variables is therefore $\mathbf{X} = \mathbf{B} = [B_1, B_2, \cdots, B_7]$, and the vector of stochastic processes $\mathbf{Y}$ is empty. Since the time factor θ appears in both functions of the motion errors, the motion errors are still stochastic processes. The motion errors should not be large, and their allowable values are denoted by $e_1 = 1.4^o$ and $e_2 = 1.4^o$. All the parameters are given in Table 3.

Table 3 Parameters in Example 2

Variable	Mean	Standard deviation	Distribution
B_1	100 mm	0.3 mm	Normal
B_2	55.5 mm	0.05 mm	Normal
B_3	144.1 mm	0.05 mm	Normal
B_4	72.5 mm	0.05 mm	Normal
B_5	79.5 mm	0.05 mm	Normal
B_6	203 mm	0.05 mm	Normal
B_7	150.8 mm	0.05 mm	Normal

The mechanism system is designed to perform its intended functions over an interval of $[\theta_0, \theta_s] = [45^\circ, 105^\circ]$. If either motion error is greater than its allowable value over $[\theta_0, \theta_s] = [45^\circ, 105^\circ]$, a failure is considered. As a result, the system is a series system, and the system probability of failure is

$$p_{f,s}(\theta_0, \theta_s) = \Pr\left\{\varepsilon_\kappa(\mathbf{B}, \chi) > e_1 \cup \varepsilon_\eta(\mathbf{B}, \tau) > e_2, \exists \chi, \tau \in [\theta_0, \theta_s]\right\} \tag{66}$$

Figs. 9 through 11 show the results of joint upcrossing rates $v_{12}^{+-}(\theta)$, $v_{12}^{-+}(\theta)$, and outcrossing rate $v_{1\cup 2}^{+}(\theta)$, from the proposed method and MCS. The sample size of MCS is 10^7.

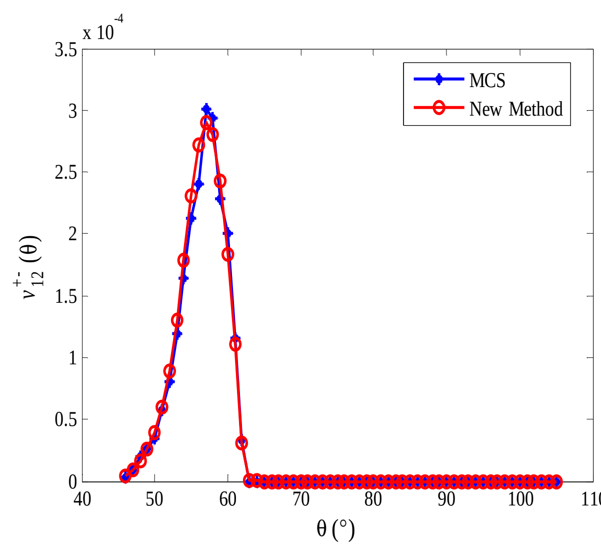

Fig. 9 $v_{12}^{+-}(\theta)$ over [45°, 105°]

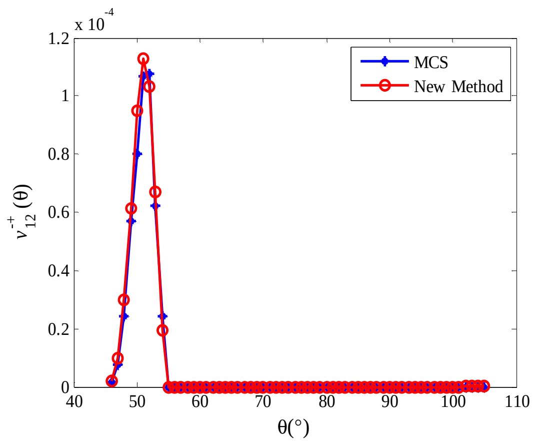

Fig. 10 $v_{12}^{-+}(\theta)$ over [45°, 105°]

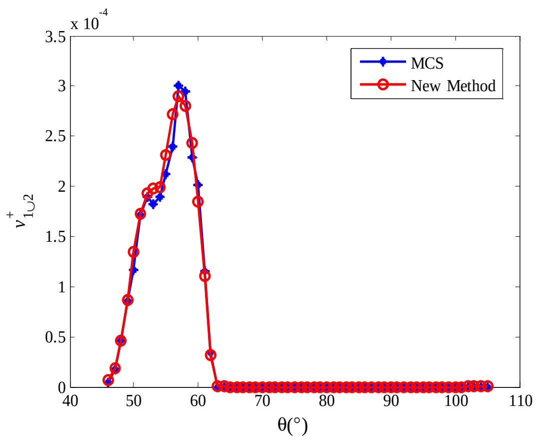

Fig. 11 $v_{1\cup 2}^{+}(\theta)$ over [45°, 105°]

The results show that the proposed method is able to estimate the joint upcrossing rate with good accuracy. Based on the joint upcrossing rates, the time-dependent system probability of failure is obtained as presented in Fig. 12 and Table 4.

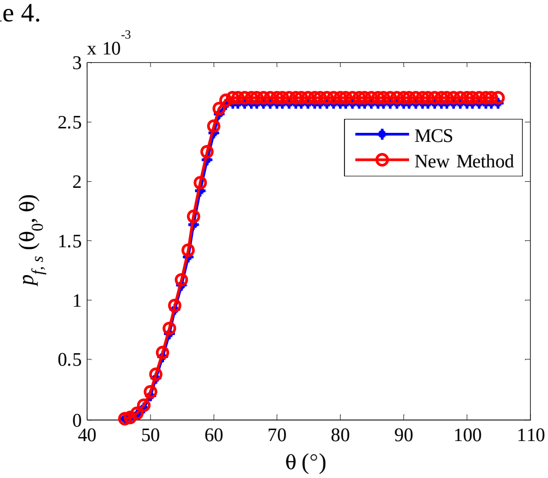

Fig. 12 Time-dependent system probability of failure over $[45^{\circ}, 105^{\circ}]$

Table 4 System time-dependent probability of failure

Time interval	New Method		MCS	
	$p_{f,s}(\theta_0,\theta)$ (10^{-3})	ε(%)	$p_{f,s}(\theta_0,\theta)$ (10^{-3})	95% confidence interval (10^{-3})
[45, 50]	0.224	13.13	0.198	[0.189, 0.206]
[45, 55]	1.165	3.56	1.125	[1.104, 1.146]
[45, 57]	1.701	3.85	1.638	[1.613, 1.663]
[45, 59]	2.253	3.16	2.184	[2.155, 2.213]
[45, 61]	2.615	2.03	2.563	[2.532, 2.595]
[45, 63]	2.694	1.32	2.659	[2.627, 2.691]
[45, 65]	2.695	1.35	2.659	[2.627, 2.691]

The results show that the accuracy of the proposed method is good.

6. CONCLUSIONS

Time-dependent system reliability analysis plays a vital role in the system level optimization, lifecycle cost estimation, and decision making on maintenance and warranty. With the availability of computational models, predicting how the system reliability changes with time is possible. Making such a prediction both accurate and efficient is critical.

In this paper, a time-dependent reliability method is proposed for a system with two response variables that are functions of random variables and stochastic processes. The method is based the First Order Reliability Method (FORM) and the upcrossing rate method (the Rice's formula). The new method can be applied to general problems with random variables, stochastic processes, and time because it can be extended to systems with more than two response variables. With the use of FORM, the proposed method is also efficient.

As an upcrossing rate method, the new method may produce a larger error if the probability of failure is larger. The reason is that when the probability of failure is large, the dependency between upcrossings may become strong.

The future research based on this work may be (1) the improvement of the accuracy, (2) the extension of the method to systems with more than two response variables, and (3) the integration of the method with optimization so that the time-dependent system reliability-based design can be performed.

APPENDIX A: DERIVATION OF $v_{ij}^{++}(t)$

Since $Z_i(t+\Delta t)>0$ and $Z_j(t+\Delta t)>0$, expanding Z_i and Z_j at time t, it gives $Z_i(t+\Delta t)\approx z_i(t)+\dot{z}_i(t)\Delta t>0$ and $Z_j(t+\Delta t)\approx z_j(t)+\dot{z}_j(t)\Delta t>0$, then Eq. (49) becomes

$$M_{ij}^{++}=\int_0^{+\infty}\int_0^{+\infty}\int_{-\dot{z}_j(t)\Delta t}^{0}\int_{-\dot{z}_i(t)\Delta t}^{0} f_{Z_iZ_j\dot{Z}_i\dot{Z}_j}(z_i,z_j,\dot{z}_i,\dot{z}_j)dz_idz_jd\dot{z}_id\dot{z}_j \quad (A1)$$

Let $Z_i(t)/(-\dot{Z}_i(t)\Delta t)=W_i(t)$ and $Z_j(t)/(-\dot{Z}_j(t)\Delta t)=W_j(t)$, Eq. (A1) is further transformed into

$$M_{ij}^{++} = \int_0^{+\infty}\int_0^{+\infty}\int_1^0\int_1^0 f_{W_iW_j\dot{Z}_i\dot{Z}_j}(-\dot{z}_i\Delta t w_i, -\dot{z}_j\Delta t w_j, \dot{z}_i, \dot{z}_j) \dot{z}_i\dot{z}_j\Delta t\Delta t dw_i dw_j d\dot{z}_i d\dot{z}_j \quad \text{(A2)}$$

Plugging Eq. (A2) into (47), we have

$$\begin{aligned} &v_{ij}^{++}(t) \\ &= \lim_{\Delta t\to 0}\frac{\int_0^{+\infty}\int_0^{+\infty}\int_1^0\int_1^0 f_{W_iW_j\dot{Z}_i\dot{Z}_j}(0, 0, \dot{z}_i, \dot{z}_j)\dot{z}_i\dot{z}_j\Delta t\Delta t dw_i dw_j d\dot{z}_i d\dot{z}_j}{\Delta t} \\ &= \lim_{\Delta t\to 0}\Delta t\int_{\dot{\beta}_j(t)}^{+\infty}\int_{\dot{\beta}_i(t)}^{+\infty} f_{L_iL_j\dot{L}_i\dot{L}_j}(\beta_i, \beta_j, \dot{l}_i, \dot{l}_j)(\dot{l}_i - \dot{\beta}_i)(\dot{l}_j - \dot{\beta}_j)d\dot{l}_i d\dot{l}_j \\ &= 0 \end{aligned} \quad \text{(A3)}$$

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